数理解析研究所講究録
736巻 1990年 27-60

Robustness Issues of Learning Control for Robotic Motions

Suguru Arimoto

Department of Mathematical Engineering
and Information Physics
Faculty of Engineering
University of Tokyo
Bunkyo-ku, Tokyo, 113 Japan

Abstract

A clss of simple learning control algorithms with a forgetting factor and a long-term memory and without use of the derivative of velocity signals is proposed for motion control of robot manipulators. The robustnes of such learning laws with respect to initialization errors, fluctuations of the dynamics, and measurement noises is studied extensively. As a result the uniform boundedness of motion trajectories is proved based on the passivity analysis of robot dynamics. It is also proved that motion trajectories converge to a neighborhood of the desired one and eventually remain in it provided the content of the long-term memory is refreshed adequately every after a sufficient number of trials.

I. Introduction

Amongst all creation man is the most skillful with his fingers and hands. We humans owe such skilled motions to inherent abilities of learning. Our babies are so clumasy with their hands that they are unable to manipulate a knife and fork. But they are able to improve their motions from repeated excercise.

The above observation led recently to the proposal of learning control theory for improvement of robot motions [1]~[9]. The concept of learning control differs from that of conventional control methodology. It is a discipline for a class of mechanical robots and mechatronics systems, which is based on autonomous self-training. It relies on the repeatability of operation characteristic to present industrial robots. The ideal principles that underlie the proposed concept of learning control are summarized as a set of postulates in the following way [10]:

A_1) Each operation ends in a finite time duration $T>0$.

A_2) A desired output $y_d(t)$ is given a-priori over that time duration $t\in[0,T]$.

A_3) Repeatability of the initialization is satisfied throughout repeated trainings, namely, the initial state $x_k(0)$ of the system can be set the same at the beginning of each operation in such a manner as

$$x_k(0) = x^0 \qquad \text{for } k = 1,2,\ldots$$

where k denotes the trial number of operation.

A_4) Invariance of the system dynamics is assured throughout repeated trainings.

A_5) Each output trajectory $y_k(t)$ can be measured without noise and thereby the error signal

$$e_k(t) = y_d(t) - y_k(t)$$

can be used in construction of the next command input.

A_6) The next command input $u_{k+1}(t)$ must be composed of a simple and fixed recursive law

$$u_{k+1} = F(u_k(t), e_k(t)).$$

For the purpose of easy implementation of the learning law, the simpler the recursive form in A_6), the better it is. However, it is implicitly expected that the recursive updation law given in A_6) gives rise to the convergence of motion trajectories, namely, outputs $y_k(t)$ approach the desired one $y_d(t)$ in some sense as k increases.

In case of motion-trajectory tracking of robot manipulators, two types of recursive updation were proposed by the author and his colleagues [1]~[9], which are described as follows (see Fig.1 and Fig. 2):

$$u_{k+1}(t) = u_k(t) + \Gamma\frac{d}{dt}(y_d(t)-y_k(t)), \tag{1}$$

$$u_{k+1}(t) = u_k(t) + \Phi(y_d(t)-y_k(t)). \tag{2}$$

In both cases both measured output y_k and desired output y_d stand for velocity signals of joint coordinates, and Γ and Φ are constant gain matrices. It has been shown in [1]~[4] that the D-type learning control defined by eq.(1) with an appropriate constant gain matrix Γ is convergent in a sense that the output trajectory approaches the desired one with the repetition of operation, namely, $y_k(t) \to y_d(t)$ uniformly in $t \in [0,T]$ as $k \to \infty$. A similar but weaker result on convergence of the P-type learning control described by eq.(2) has also been obtained by Arimoto and Kawamura [5], [7] and [8], but the argument in its proof is based on a linearized dynamics model of the robot manipulator around the desired motion trajectory and the ignorance of higher terms. In addition, it was implicitly assumed in both cases that the manipulator must take the same initial position and velocity at every operation trial and none of fluctuations of dynamics and measurement noise arises, namely, postulates from A_3) to A_5) must be satisfied throughout the repetition of exercise. Although most of present industrial robots may satisfy approximately these postulates because of their superiority of repeatability precision, it is crucial to assure the technical soundness and robustness of the learning control with respect to small but persistent errors of initialization, fluctuations of dynamics, and measurement noise during operation. In other words, it is important to relax conditions posed in postulates A_3)~A_5) to some extent in the following way:

A_3') Repeatability of the initialization is satisfied within an admissible level of deviation, i.e., the initial state $x_k(0)$ of the system can be set as follows:

$$x_k(0) = x^0 = \delta_k, \quad |\delta_k| < \varepsilon_1 \tag{3}$$

for some $\varepsilon_1>0$, where the norm $|x|$ of vector $x=(x^1,\dots,x^n)^T$ is defined in this paper as

$$|x| = \max_{i=1,\dots,n} |x^i|.$$

A_4') Fluctuations $\eta_k(t)$ which may appear in robot dynamics must astisfy

$$\|\eta_k\|_\infty \leqq \varepsilon_2 \tag{4}$$

for some $\varepsilon_2>0$, where the function norm is defined as

$$\|\eta_k\|_\infty = \sup_{t\in[0,T]} |\eta_k|.$$

A_5') Each output trajectory $y_k(t)$ can be measered within a small specified noise level, i.e.,

$$e_k(t) = y_d(t) - (y_k(t)+\xi_k(t)) \tag{5}$$

where ξ_k must satisfy

$$\|\xi_k\|_\infty \leqq \varepsilon_3 \tag{6}$$

for some $\varepsilon_3>0$.

The robustness problem in which postulaters $A_3')\sim A_5')$ are assumed instead of $A_3)\sim A_5)$ was first discussed by Arimoto et al [10] in case of the PID-type learning scheme under an assumption that the initial trajectory (and thus all subsequent ones) lies in a neighborhood of the desired trajectory and hence the robot dynamics can be considered to be subject to a linearized madel (a linear time-varying mechanical system). Very recently, Heinzinger et al [11] attacked the same robustness problem for a class of D-type learning control and proved without use of any linearization that the learned input and the corresponding output trajectories converge to neighborhoods of their desired ones. They also made a comment by illustrating a counter-example that such a robustness property is not valid for a class of PI-type learning.

In the present paper we introduce a forgetting factor $\alpha>0$ into PI-type learning schemes in the following manner (see Fig.3):

$$u_{k+1}(t) = (1-\alpha)u_k(t) + \Phi\{y_d(t)-(y_k(t)+\xi_k(t))\}. \qquad (7)$$

We study in details robstness problems of such a learning control with respect to initialization errors, fluctuations of dynamics, and measurement noise. The original idea of use of the forgetting factor into learning schemes is due to Heinzinger et al [11], but it was introduced into only D-type learning. In a previous paper [12] we proved with aid of linearization that even in case of P-type learning control schemes with a forgetting factor the motion trajectories converge to a neighborhood of the desired one and eventually remain in it. This paper pproves this under full robot dynamics with nonlinearity by fully using the following two basic characteristics inherent to robot dynamics: 1) Generalized passivity of the joint velocity vector (output) with respect to the torque input vector and 2) Existence of an energy-like Lyapunov function weighted exponentially. Furthermore, we show that the size of attraction neighborhoods can be chosen small dependently on the magnitudes of initialization errors and other desturbances. Hence, if an initial input u_0 is chosen carefully and put into a long-term memory as in Fig.4 and the updation law

$$\begin{aligned} u_{k+1}(t) = {} & (1-\alpha)u_k(t) + \alpha u_0(t) \\ & + \Phi\{y_d(t)-(y_k(t)+\xi_k(t))\} \end{aligned} \tag{8}$$

is adopted, then the refinement of trajectories becomes noteworthy.

In the next section we present basic inequalities deduced from the generalized passivity valid for dynamics of displacement joint vector $r_k(f) = q_k(t)+q_d(t)$. The uniform boundedness of a succession of trajectories through repeated trainings follows immediately from these inequalities. They also play a vital role in the proof of convergence of the learning scheme.

II. Generalized Passivity of Robot Dynamics and Uniform Boundedness of Trajectories.

When the velocity signals $y_k(t)$ at joints are measured, numerical differentiation is inevitable in digital implementation of D-type learning schemes. This may cause an additional noise in input updation in eq.(1) if $y_k(t)$ is contaminated with noise. Although this effect can be reduced to some extent by employing an adequate digital filter that approximates the operator of differentiation as pointed out by Atkeson and McIntyre [13], it is reasonable to avoid further troublesome operation. In reality, many experimental results reported by the author and his colleagues [8] [9] have shown that the P-type learning algorithm without use of differentiation works well in tracking given joint trajectories. However, theoretical treatments of the case of P-type learning are considerably difficult in comparison with the case of D-type learning. Our previous proof of convergence of the P-type learning algorithm found in [5] and [9] is far from strong as pointed out in the following : 1) It was based upon a linearized madel of robot dynamics around the given desired trajectory, 2) the remaining terms of higher order were ignored in the model, and 3) the uniform boundedness of velocity trajectories for consecutive trials k was implicitly assumed. To clear all these defects of the proof, it is vital to gain an insight into the basic characteristics of dynamics inherent to robotic manipulators.

In this paper we consider a class of serial-link manipulators of all revolute-type joints. It is then well-known that the dynamics of such a monipulator can be described in terms of the joint coordinates vector q in the following way:

$$(J+H(q))\ddot{q} + (B_0+\dot{H}(q))\dot{q} - \frac{\partial T}{\partial q} + g(q) = Kv. \tag{9}$$

We also suppose that a linear PD feedback servo

$$v = u + K_0(q_d-q) - K_1\dot{q} \tag{10}$$

is employed as an inner control loop. In these equations, q_d is a given desired command input, H an inertia matrix, J a positive diagonal matrix representing inertial terms of internal load distribution of actuators, $T=\dot{q}^T(J+H(q))\dot{q}/2$ the kinetic energy, g(q) a vector of gravity terms, v a vector of input voltages or currents to actuators, B_0 a positive definite matrix representing damping factors and coefficients of electro-motive forces, and K, K_0 and K_1 diagonal matrices of positive gains respectively. Then, it is possible to assume that at the k-th trial of operation the robot motion is subject to

$$(J+H(q_k))\ddot{q} + (B+\dot{H}(q_k))\dot{q}_k - \frac{\partial T}{\partial q_k} + g(q_k) + A(q_k-q_d) = u_k + \eta_k \tag{11}$$

where Ku_k is rewritten into u_k renewedly without loss of generality, η_k refers to the fluctuation term mentioned in Postulate A_4') previously, and

$$B = B_0+KK_1, \qquad A = KK_0. \tag{12}$$

Note that the inertia matrix $H(q)$ is symmetric and positive definite and, moreover, each entry of H is constant or a trigonometric function in components of joint vector q. Hence, $H(q)$ and any of partial derivatives of $H(q)$ with respect to q^i are uniformly Lipschitz continuous in q. Next observe that eq.(11) can be written in the form

$$(J+H(q_k))\ddot{q}_k + (B+\tfrac{1}{2}\dot{H}(q_k))\dot{q}_k + C(q_k,\dot{q}_k)\dot{q}_k + g(q_k) + A(q_k-q_d) = u_k + \eta_k \tag{13}$$

where

$$C(q_k,\dot{q}_k)\dot{q}_k = \tfrac{1}{2}\dot{H}(q_k)\dot{q}_k - \frac{\partial}{\partial q_k}\{\tfrac{1}{2}\dot{q}_k^T H(q_k)\dot{q}_k\}. \tag{14}$$

As was pointed out first by Arimoto and Miyazaki [14] [15] and later but independently by Koditschek [16], $C(q_k,\dot{q}_k)$ is skew symmetric, in other words,

$$\dot{q}^T C(q,\dot{q})\dot{q} = 0 \tag{15}$$

It should be also noted that the invertibility of robot dynamics implies the existence of the unique desired input $u_d(t)$ for a given desired output $y_d(t)(=\dot{q}_d(t))$ provided that $y_d(t)$ is differentiable and $\dot{y}_d(t)$ is piecewise continuous on $t\in[0,T]$. This yields

$$(J+H(q_d))\ddot{q}_d + (B+\frac{1}{2}\dot{H}(q_d))\dot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + g(q_d) = u_d. \quad (16)$$

Then, it is convenient to introduce the displacement vector $r_k = q_k - q_d$ and important to see that r_k satisfies the equation

$$\begin{aligned}(J+H(q_k))\ddot{r}_k + (B+\frac{1}{2}\dot{H}(q_k))\dot{r}_k + C(q_k,\dot{q}_k)\dot{r}_k \\ + Ar_k + f_k = \Delta u_k + \eta_k \end{aligned} \quad (17)$$

where

$$\begin{aligned}&\Delta u_k = u_k - u_d, \quad r_k = q_k - q_d, \\ &f_k = f(r_k,\dot{r}_k) = \{H(q_d+r_k)-H(q_d)\}\ddot{q}_d + \frac{1}{2}\{\dot{H}(q_d+r_k)-\dot{H}(q_d)\}\dot{q}_d \\ &\quad + \{C(q_d+r_k,\dot{q}_d+\dot{r}_k)-C(q_d,\dot{q}_d)\}\dot{q}_d + g(q_d+r_k) - g(q_d)\end{aligned} \quad (18)$$

Note that f_k can be written in the form

$$\begin{aligned}f_k = E_k(q_d,\dot{q}_d,\ddot{q}_d)r_k + F_k(q_d,\dot{q}_d,\ddot{q}_d)\dot{r}_k \\ + h_k(q_d,\dot{q}_d,\ddot{q}_d,r_k,\dot{r}_k)\end{aligned} \quad (19)$$

where h_k represents the remaining higher terms of r_k and $\dot{r}_k$.

Now, remind that one of the basic characteristics of robot dynamics (for example, eq.(9)) is the passivity of the velocity output with respect to the actuator input. A similar result must be obtained by taking the inner product of vectors $\dot{r}_k$ and Δu_k in eq.(17) and taking the integration over [0,T]. However, for the purpose of proving the convergence of P-type learning, we need a slightly generalized and more powerful result, which can be obtained by taking the exponentially weighted inner product of $\dot{r}_k$ and Δu_k in eq.(17). This is done in the following manner:

$$
\begin{aligned}
&\int_0^t e^{-\lambda\tau}\dot{r}_k^T\Delta u_k d\tau \\
&= \int_0^t e^{-\lambda\tau}\dot{r}_k^T[(J+H(q_k))\ddot{r}_k+(B+\tfrac{1}{2}\dot{H}(q_k))\dot{r}_k \\
&\qquad + C(q_k,\dot{q}_k)\dot{r}_k+Ar_k+f_k-\eta_k]d\tau \\
&= \frac{1}{2}\int_0^t \frac{d}{d\tau}[e^{-\lambda\tau}\{\dot{r}_k^T(J+H(q_k))\dot{r}_k+r_k^TAr_k\}]d\tau \\
&\qquad + \int_0^t e^{-\lambda\tau}\dot{r}_k^TB\dot{r}_k d\tau + \int_0^t e^{-\lambda\tau}\dot{r}_k^Tf_k d\tau - \int_0^t e^{-\lambda\tau}\dot{r}_k^T\eta_k d\tau \\
&\qquad + \frac{1}{2}\int_0^t \lambda e^{-\lambda\tau}\{\dot{r}_k^T(J+H(q_k(t))\}\dot{r}_k+r_k^TAr_k\}d\tau \\
&= e^{-\lambda t}V(r_k(t),\dot{r}_k(t)) - V(r_k(0),\dot{r}_k(0)) \\
&\qquad + \int_0^t e^{-\lambda\tau}[\lambda V(r_k,\dot{r}_k)+\dot{r}_k^TB\dot{r}_k]d\tau \\
&\qquad + \int_0^t e^{-\lambda\tau}\dot{r}_k^T(f_k-\eta_k)d\tau, \qquad (20)
\end{aligned}
$$

where

$$V(r_k,\dot{r}_k) = \frac{1}{2}\{\dot{r}_k^T(J+H(q_k))\dot{r}_k+r_k^TAr_k\}. \qquad (21)$$

For the time being, we assume the uniform boundedness of $r_k(t)$ and $\dot{r}_k(t)$ in k, which will be eventually proved in this section. In other words, there are two neighborhoods N_1 and N_2 defined by

$$N_1(\gamma_1) = \{r(t):\|r(t)\|_\infty<\gamma_1\},$$
$$N_2(\gamma_2) = \{\dot{r}(t):\|\dot{r}(t)\|_\infty<\gamma_2\} \tag{22}$$

such that

$$r_k(t) \in N_1(\gamma_1), \qquad \dot{r}_k(t) \in N_2(\gamma_2) \tag{23}$$

for all k. Then, from the uniform Lipschitz continuity of $H(q)$, $\partial H(q)/\partial q^i$, and $g(q)$, there are constants $\rho_0>0$, $\rho_1>0$, and $c_2>0$ such that

$$\begin{aligned} |\dot{r}_k^T(f_k-\eta_k)| &\leqq |\dot{r}_k^T E_k r_k| + |\dot{r}_k^T F_k \dot{r}_k| + |\dot{r}_k^T h_k| + |\dot{r}_k^T \eta_k| \\ &\leqq \rho_0 r_k^T r_k + \rho_1 \dot{r}_k^T \dot{r}_k + c_2 \varepsilon_2 \end{aligned} \tag{24}$$

for all $r_k \in N_1(\gamma_1)$ and $\dot{r}_k \in N_2(\gamma_2)$. Substituting this inequality into eq. (20) yields

$$\begin{aligned} \int_0^t e^{-\lambda\tau}\dot{r}_k^T \Delta u_k d\tau &\geqq e^{-\lambda t} V(r_k(t),\dot{r}_k(t)) \\ &- V(r_k(o),\dot{r}_k(o)) + \int_0^t e^{-\lambda\tau} W(\lambda;r_k(\tau),\dot{r}_k(\tau))d\tau - c_2\varepsilon_2, \end{aligned} \tag{25}$$

where

$$W(\lambda;r_k,\dot{r}_k) = \frac{1}{2}r_k^T(\lambda A-2\rho_0 I)r_k + \frac{1}{2}\dot{r}_k^T[\lambda\{J+H(q_k)\}+2B-2\rho_1 I]\dot{r}_k. \tag{26}$$

Now we choose $\lambda>0$ large enough so that $W(\lambda;r_k,\dot{r}_k)$ is positive definite in r_k and $\dot{r}_k$. Then it follows that

$$\int_0^t e^{-\lambda\tau}\dot{r}_k \Delta u_k d\tau \geqq - V(r_k(o),\dot{r}_k(o)) - c_2\varepsilon_2 \tag{27}$$

which shows the generalized passivity of velocity displacement $\dot{r}_k$ with respect to input Δu_k for the displacement dynamics described by eq.(17).

Now we are in a position to show the uniform boundedess of trajectories for the P-type learning algorithm described by eq.(7) or more generally by eq.(8). To do this, first consider eq.(13) for the case of k=0.

<u>Lemma 1</u> If $0<\Phi\leqq 2B$ and $u_0(t)$ satisfies the inequality

$$\int_0^t (u_o-u_d)^T\Phi^{-1}(u_o-u_d)d\tau \leqq \gamma_0, \tag{28}$$

then there are constants $\beta_1>0$ and $\beta_2>0$ depending on ε_1, ε_2, and γ_0 such that

$$\|q_k-q_d\|_\infty \leqq \beta_1 \quad \text{and} \quad \|\dot{q}_k-\dot{q}_d\|_\infty \leqq \beta_2. \tag{29}$$

<u>Proof</u> Taking the inner product of $\dot{q}_0$ with u_0 via eq.(13) when k=0, we obtain

$$\int_0^t \dot{q}_0^T u_0 d\tau = V(q_0(t),\dot{q}_0(t)) - V(q_0(0),\dot{q}_0(0))$$
$$+ G(q_0(t)) - G(q_0(0)) + \int_0^t \dot{q}_0^T(B\dot{q}_0-Aq_d-\eta_0)d\tau \tag{30}$$

where $G(q)$ is the potential energy for the manipulator from which the gravity term $g(q)$ is generated, namely, $\partial G(q)/\partial q^i = g^i(q)$. Substitution of inequalities

$$\int_0^t \dot{q}_0^T (u_0 + Aq_d + \eta_0) d\tau \leqq \frac{1}{2}\int_0^t \dot{q}_0^T \Phi \dot{q}_0 d\tau + \int_0^t (u_0 + Aq_d + \eta_0)^T \Phi^{-1} (u_0 + Aq_d + \eta_0) d\tau,$$

$$\frac{1}{2}\int_0^t (u_0 + Aq_d + \eta_0)^T \Phi^{-1} (u_0 + Aq_d + \eta_0) d\tau \leqq \int_0^t (u_0 - u_d)^T \Phi^{-1} (u_0 - u_d) d\tau + \int_0^t (u_d + Aq_d + \eta_0)^T \Phi^{-1} (u_d + Aq_d + \eta_0) d\tau$$

into eq.(30) subsequently gives rise to

$$\begin{aligned} V(q_0(t), \dot{q}_0(t)) + G(q_0(t)) &\leqq V(q_0(0), \dot{q}_0(0)) + G(q_0(0)) \\ &\quad - \int_0^t \dot{q}_0^T (B - \tfrac{1}{2}\Phi) \dot{q}_0 d\tau + \int_0^t (u_0 - u_d) \Phi^{-1} (u_0 - u_d) d\tau \\ &\quad + \int_0^t (u_d + Aq_d + \eta_0)^T \Phi^{-1} (u_d + Aq_d + \eta_0) d\tau \\ &\leqq \gamma_0 + V(q_0(0), \dot{q}_0(0)) + G(q_0(0)) \\ &\quad + \int_0^T (u_d + Aq_d + \eta_0)^T \Phi^{-1} (u_d + Aq_d + \eta_0) d\tau \end{aligned}$$

Since the right hand side of this equation is bounded from above, $V(q_0, \dot{q}_0) + G(q_0)$ is also bounded from above. Since again $V(q_0, \dot{q}_0)$ is positive definite in q_0 and $\dot{q}_0$ as defined by eq.(21) and $G(q_0)$ is a periodic function of q_0, both $q_0(t)$ and $\dot{q}_0(t)$ must be bounded, which implies the existence of β_1 and β_2 such that eq.(29) is satisfied.

We are now going to prove the uniform boundedness of position and velocity trajectories for a class of P-type learning control algorithms when errors of initialization, measurement noises and fluctuations of system dynamics may arise to some extent.

Suppose that the system dynamics, the measurement process, and the learning law are subject to eqs.(13), (15), and (8) respectively. First we take an appropriate constant $\gamma>0$, which need not be small and call a control input u_0 "admissible" if it is piecewise continuous, $x_0(0)=(q_0(0)-q_d(0),\dot{q}_0(0)-\dot{q}_d(0))$ satisfies eq.(3), and satisfies

$$\int_0^T (u_0-u_d)^T\Phi^{-1}(u_0-u_d)d\tau < \gamma. \tag{31}$$

For a given admissible $u_0(t)$, denote by $q_0(t)$ the solution to the system of eq.(13) at k=0. We also call such a solution $q_0(t)$ admissible. Next define

$$\gamma_1 = \sup\|q_0-q_d\|_\infty, \quad \gamma_2 = \sup\|\dot{q}_0-\dot{q}_d\|_\infty \tag{32}$$

where supremum is taken over all such admissible solutions. The existence of such upper bounds is assured by Lemma 1. Then, according to the form of f_k described by eq.(18), there are constants $\rho_0>0$ and $\rho_1>0$ such that

$$|\dot{r}_0^T f(r_0,\dot{r}_0)| \leqq \rho_0 r_0^T r_0 + \rho_1 \dot{r}_0^T \dot{r}_0 \tag{33}$$

for all aemissible solutions q_0 as in eq.(24), where $r_0=q_0-q_d$ and $\dot{r}_0=\dot{q}_0-\dot{q}_d$. Next we choose a small $\lambda>0$ so that inequalities

$$(1-\alpha)\{2B-2\rho_1 I+\lambda(J+H(q))\} - \Phi > 0, \tag{34}$$

$$\lambda A - 2\rho_0 I > 0 \tag{35}$$

for all q and fix it for ever. Note that these inequalities of matrices are similar to the positive definiteness of the quadratic form appeared in eq.(26) if $\Phi=0$ and $\alpha=0$.

Now, it is important to note that since by definition

$$\Delta u_{k+1} = (1-\alpha)\Delta u_k + \alpha\Delta u_0 + \Phi e_k \tag{36}$$

it holds

$$\begin{aligned}\int_0^t & e^{-\lambda\tau}(\Delta u_{k+1}-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_{k+1}-\alpha\Delta u_0)d\tau \\ &= \int_0^t e^{-\lambda\tau}e_k^T\Phi e_k d\tau + 2(1-\alpha)\int_0^t e^{-\lambda\tau}\Delta u_k^T e_k d\tau \\ &\quad + (1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_k^T\Phi^{-1}\Delta u_k d\tau.\end{aligned} \tag{37}$$

We observe that the last term of the right hand side can be bounded from above in the following way:

$$\begin{aligned}(1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_k^T\Phi^{-1}\Delta u_k d\tau &= \int_0^t e^{-\lambda\tau}[(1-\alpha)(\Delta u_k-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_k-\alpha\Delta u_0) \\ &+ \alpha(1-\alpha)^2\Delta u_0^T\Phi^{-1}\Delta u_0-\alpha(1-\alpha)(\Delta u_k-\Delta u_0)^T\Phi^{-1}(\Delta u_k-\Delta u_0)]d\tau\end{aligned}$$

$$\leqq \int_0^t e^{-\lambda\tau}[(1-\alpha)(\Delta u_k-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_k-\alpha\Delta u_0)$$
$$+ \alpha(1-\alpha)^2\Delta u_0^T\Phi^{-1}\Delta u_0]d\tau. \quad (38)$$

By using eq.(20), the second term can be rewritten in the following way:

$$- \int_0^t e^{-\lambda\tau}\Delta u_k^T e_k d\tau = \int_0^t e^{-\lambda\tau}\Delta u_k^T(\dot{r}_k+\xi_k)d\tau = e^{-\lambda t}V(r_k(t),\dot{r}_k(t))$$
$$+ \int_0^t e^{-\lambda\tau}[\lambda V(r_k,\dot{r}_k)+\dot{r}_k^T B\dot{r}_k]d\tau - V(r_k(0),\dot{r}_k(0))$$
$$+ \int_0^t e^{-\lambda\tau}\dot{r}_k^T(f_k-\eta_k)d\tau + \int_0^t e^{-\lambda\tau}\Delta u_k^T\xi_k d\tau \quad (39)$$

Substituting eqs.(38) and (39) into eq.(37) gives rise to

$$\int_0^t e^{-\lambda\tau}(\Delta u_{k+1}-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_k-\alpha\Delta u_0)d\tau$$
$$\leqq (1-\alpha)\int_0^t e^{-\lambda\tau}(\Delta u_k-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_k-\alpha\Delta u_0)d\tau$$
$$- 2(1-\alpha)e^{-\lambda t}V(r_k(t),\dot{r}_k(t))$$
$$- \int_0^t e^{-\lambda\tau}[2(1-\alpha)\lambda V(r_k,\dot{r}_k)+2(1-\alpha)\dot{r}_k^T B\dot{r}_k-\dot{r}_k\Phi\dot{r}_k]d\tau$$
$$- 2(1-\alpha)\int_0^t e^{-\lambda\tau}\dot{r}_k^T(f_k-\eta_k)d\tau$$
$$+ 2(1-\alpha)V(r_k(0),\dot{r}_k(0))-2(1-\alpha)\int_0^t e^{-\lambda\tau}\Delta u_k^T\xi_k d\tau$$
$$+ \alpha(1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_0^T\Phi^{-1}\Delta u_0 d\tau$$
$$+ 2\int_0^t e^{-\lambda\tau}\dot{r}_k^T\Phi\xi_k d\tau + \int_0^t e^{-\lambda\tau}\xi_k^T\Phi\xi_k d\tau. \quad (40)$$

At this stage, we bear in mind that, in accordance with postulates $A_3') \sim A_5')$, $x_k(0)=(r_k(0),\dot{r}_k(0))$, $\eta_k(t)$ and $\xi_k(t)$ satisfy eqs.(3), (4) and (6) respectively. Hence, in particular for k=0, there exist positive constants c_1, c_2, and c_3 such that

$$
\begin{aligned}
&V(r_0(0),\dot{r}_0(0)) \leqq c_1\varepsilon_1^2, \\
&\int_0^T e^{-\lambda\tau}|\dot{r}_0^T\eta_0|d\tau \leqq c_2\varepsilon_2, \\
&\int_0^T e^{-\lambda\tau}\{2|\Delta u_0^T\xi_0|+2|\dot{r}_0^T\Phi\xi_0|+|\xi_0^T\Phi\xi_0|\}d\tau \leqq c_3\varepsilon_3
\end{aligned}
\tag{41}
$$

for all admissible $q_0(t)$. Substituting these inequalities and eq.(33) into eq.(40) at k=0, we obtain

$$
\begin{aligned}
&\int_0^t e^{-\lambda\tau}(\Delta u_1-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_1-\alpha\Delta u_0)d\tau \\
&\quad \leqq (1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_0^T\Phi^{-1}\Delta u_0 d\tau \\
&\qquad - 2(1-\alpha)e^{-\lambda t}V(r_0(t),\dot{r}_0(t)) \\
&\qquad - \int_0^t e^{-\lambda\tau}U(\lambda;r_0,\dot{r}_0)d\tau \\
&\qquad + 2c_1\varepsilon_1^2 + 2c_2\varepsilon_2 + c_3\varepsilon_3
\end{aligned}
\tag{42}
$$

where we define in general

$$
\begin{aligned}
U(\lambda;r_k,\dot{r}_k) &= 2(1-\alpha)\lambda V(r_k,\dot{r}_k) + 2(1-\alpha)\dot{r}_k B\dot{r}_k \\
&\quad - \dot{r}_k\Phi\dot{r}_k - 2(1-\alpha)\rho_0 r_k^T r_k - 2(1-\alpha)\rho_1\dot{r}_k^T\dot{r}_k \\
&= \dot{r}_k^T[(1-\alpha)\{2B-2\rho_1 I+\lambda(J+H(r_k+q_d))\}-\Phi]\dot{r}_k \\
&\quad + (1-\alpha)r_k[\lambda A-2\rho_0 I]r_k.
\end{aligned}
\tag{43}
$$

Since $U(\lambda;r_k,\dot{r}_k)$ is positive definite according to eqs.(34) and (35) and in particular $U(\lambda;r_0,\dot{r}_0)$ is so, we have

$$\int_0^t e^{-\lambda t}(\Delta u_1-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_1-\alpha\Delta u_0)d\tau \\ \leqq (1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_0^T\Phi^{-1}\Delta u_0 d\tau + \varepsilon \qquad (44)$$

where

$$\varepsilon = 2c_1\varepsilon_1^2 + 2c_2\varepsilon_2 + c_3\varepsilon_3. \qquad (45)$$

Finally, note that in general

$$\begin{aligned}(\Delta u_k-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_k-\alpha\Delta u_0) \\ = \tfrac{1}{2}\Delta u_k^T\Phi^{-1}\Delta u_k - \alpha^2\Delta u_0^T\Phi^{-1}\Delta u_0 \\ + \tfrac{1}{2}(\Delta u_k-2\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_k-2\alpha\Delta u_0) \\ \geqq \tfrac{1}{2}\Delta u_k^T\Phi^{-1}\Delta u_k - \alpha^2\Delta u_0^T\Phi^{-1}\Delta u_0\end{aligned} \qquad (46)$$

Substituting this equation at k=1 into eq.(44) yields

$$\int_0^T e^{-\lambda\tau}\Delta u_1^T\Phi^{-1}\Delta u_1 d\tau \leqq 2\{\alpha^2+(1-\alpha)^2\}\int_0^T e^{-\lambda\tau}\Delta u_0\Phi^{-1}\Delta u_0 d\tau + 2\varepsilon \qquad (47$$

Thus, if Δu_0 satisfies

$$\int_0^T e^{-\lambda\tau}\Delta u_0^T\Phi^{-1}\Delta u_0 d\tau \leqq \gamma_0 \qquad (48)$$

for a constant γ_0 such that

$$2\gamma_0 + 2\varepsilon \leqq e^{-\lambda T}\gamma, \tag{49}$$

then

$$\int_0^T \Delta u_1^T \Phi^{-1} \Delta u_1 d\tau \leqq e^{\lambda T} \int_0^T e^{-\lambda\tau} \Delta u_1^T \Phi^{-1} \Delta u_1^T d\tau \leqq \gamma. \tag{50}$$

In other words, $u_1(t)$ becomes admissible. To prove further the admissibility of $u_k(t)$ for all $k=2,3,\ldots,$ we need a more restricted condition on γ_0 than eq.(47).

Now we prove the following theorem by induction.

<u>Theorem 1</u> Assume that $2B>\Phi>0$, all ε_1, ε_2 and ε_3 in postulates $A_3') \sim A_5')$ are small, and $u_0(t)$ is piecewise continuous and satisfies

$$\int_0^T e^{-\lambda\tau}\{u_0(t)-u_d(t)\}^T \Phi^{-1}\{u_0(t)-u_d(t)\}d\tau \leqq \gamma_0 \tag{51}$$

for a costant γ_0 such that

$$2\gamma_0 + 2\varepsilon/\alpha \leqq e^{-\lambda T}\gamma. \tag{52}$$

Then all trajectories of solutions generated by the set of recursive equations (17) and (8) with initializations $x_k(0)=(q_k(0)-q_d(0),\dot{q}_k(0)-\dot{q}_d(0))$ satisfying eq.(3) are bounded uniformly in k. More detailedly, it holds

$$\|q_k-q_d\|_\infty \leqq \gamma_1, \qquad \|\dot{q}_k-\dot{q}_d\|_\infty \leqq \gamma_2 \tag{53}$$

where constants γ_1 and γ_2 are defined in eq.(32).

<u>Proof</u> The admissibility of $u_1(t)$ was already proved above. Next we assume that $u_i(t)$ for all i=2,...,k are admissible. Then, all $q_i(t)$ for i=1,...,k are admissible and hence $r_i(t)$ and $\dot{r}_i(t)$ for all i=1,...,k satisfies

$$\|r_i\|_\infty \leqq \gamma_1, \qquad \|\dot{r}_i\|_\infty \leqq \gamma_2 \qquad \text{for } i=1,\dots,n. \tag{54}$$

Then, by the same reason as in derivation of eqs.(33) and (41), we can derive the following inequalities for the same constants ρ_0, ρ_1, c_1, and c_2 as in eqs.(33) and (41):

$$\begin{aligned}
&|\dot{r}_i f(r_i,\dot{r}_i)| \leqq \rho_0 r_i^T r_i + \rho_1 \dot{r}_i^T \dot{r}_i, \\
&V(r_i(0),\dot{r}_i(0)) \leqq c_1\varepsilon_1^2, \\
&\int_0^T e^{-\lambda\tau}|\dot{r}_i^T\eta_i|d\tau \leqq c_2\varepsilon_2, \\
&\int_0^T e^{-\lambda\tau}\{2|\Delta u_i^T\xi_i|+2|\dot{r}_i^T\Phi\xi_i|+|\xi_i^T\Phi\xi_i|\}d\tau \leqq c_3\varepsilon_3.
\end{aligned} \tag{55}$$

Substituting these inequalities into eq.(40), we obtain

$$\begin{aligned}
&\int_0^t e^{-\lambda\tau}(\Delta u_{i+1}-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_{i+1}-\alpha\Delta u_0)d\tau \\
&\quad \leqq (1-\alpha)\int_0^t e^{-\lambda\tau}(\Delta u_i-\alpha\Delta u_0)\Phi^{-1}(\Delta u_i-\alpha u_0)d\tau
\end{aligned}$$

$$- 2(1-\alpha)e^{-\lambda t}V(r_i(t),\dot{r}_i(t))$$
$$- \int_0^t e^{-\lambda\tau}U(\lambda;r_i,\dot{r}_i)d\tau$$
$$+ \alpha(1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_0^T\Phi^{-1}\Delta u_0 d\tau + \varepsilon \qquad (56)$$

where ε is defined by eq.(45). Since both V and U are positive definite in r_i and $\dot{r}_i$, eq.(56) implies

$$\int_0^t e^{-\lambda\tau}(\Delta u_{k+1}-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_{k+1}-\alpha\Delta u_0)d\tau$$
$$\leqq (1-\alpha)^{k+1}(1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_0^T\Phi^{-1}\Delta u_0 d\tau$$
$$+ \frac{1-(1-\alpha)^{k+1}}{1-(1-\alpha)}[\alpha(1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_0^T\Phi^{-1}\Delta u_0 d\tau+\varepsilon]$$
$$\leqq (1-\alpha)^2\int_0^t e^{-\lambda\tau}\Delta u_0^T\Phi^{-1}\Delta u_0 d\tau + \varepsilon/\alpha. \qquad (57)$$

Applying again inequality (46) for the above inequality, we finally obtain

$$\int_0^t e^{-\lambda\tau}\Delta u_{k+1}^T\Phi^{-1}\Delta u_{k+1}d\tau \leqq 2\int_0^t e^{-\lambda\tau}\Delta u_0^T\Phi\Delta u_0 d\tau + 2\varepsilon/\alpha$$
$$\leqq 2\gamma_0 + 2\varepsilon/\alpha \leqq e^{-\lambda T}\gamma, \qquad (58)$$

which implies the admissibility of the input u_{k+1} at the k+1th trial. By induction, all $u_k(t)$ for k=1,2,..., are admissible and hence, by Lemma 1, all $q_k(t)$ and $\dot{q}_k(t)$ are bounded uniformly in k. Thus the theorem has been proved.

The problem of uniform boundeness of trajectories generated by iterative learning control schemes was first treated by Bondi et al [17] for the case of PID-type learning law for robotic motions. However, the argument employed in the paper is based upon a linearization technic around the desired trajectory. Hence, their results are valid in a local sense that the initial command input u_0 must be sufficiently close to the desired input u_d. In contrast, Theorem 1 in this paper assures the uniform boundedness for P-type learning schemes in a global sense provided all errors $\varepsilon_1 \sim \varepsilon_3$ in postulates $A_3') \sim A_5')$ are not so large as the forgetting factor α. A slightly weaker version of Thorem 1 was presented in our previous paper [18] by using a similar but a little obscure argument.

III. Convergence of Learning Control with a Forgetting Factor

We now turn to the convergence problem of motion trajectories. To solve this, eq.(56) plays a vital role together with the uniform boundedness of motion trajectories proved in Theorem 1. It is also crucial to note that eq.(56) is valid for all λ larger than the fixed λ determined at the stage of definitions of eqs.(34) and (35). Hence, by defining

$$\begin{aligned} s_i(t) &= \int_0^t e^{-(\lambda+1)\tau}(\Delta u_i-\alpha\Delta u_0)^T\Phi^{-1}(\Delta u_i-\alpha\Delta u_0)d\tau, \\ V_i(t) &= V_i(r_i(t),\dot{r}_i(t)), \\ U_i(\lambda;t) &= \int_0^t e^{-(\lambda+1)\tau}U(\lambda+1;r_i(\tau),\dot{r}_i(\tau))d\tau, \\ Q(\lambda;\Delta u_0) &= \int_0^t e^{-(\lambda+1)\tau}\Delta u_0^T\Phi^{-1}\Delta u_0(\tau)d\tau \end{aligned} \tag{59}$$

eq.(56) can be rewritten into the following form:

$$\begin{aligned} s_{i+1}(t) \leqq{} & (1-\alpha)s_i(t) - 2(1-\alpha)V_i(t) \\ & - U_i(\lambda;t) + \alpha(1-\alpha)^2Q(\lambda;\Delta u_0) + \varepsilon. \end{aligned} \tag{60}$$

The summation of this equation from i=0 to i=k-1 gives rise to

$$\begin{aligned} s_k(t) \leqq{} & s_0(t) - \alpha\sum_{i=0}^{k-1}s_i(t) - \sum_{i=0}^{k-1}\{2(1-\alpha)V_i(t)+U_i(\lambda;t)\} \\ & + k\alpha(1-\alpha)^2Q(\lambda;\Delta u_0) + k\varepsilon. \end{aligned} \tag{61}$$

Dividing this by k and omitting the second term of the right hand side since $s_i(t) \geqq 0$ for all i, we obtain

$$\frac{1}{k}\sum_{i=0}^{k-1}\{2(1-\alpha)V_i(t)+U_i(\lambda;t)\} \leqq \alpha(1-\alpha)^2 Q(\lambda;\Delta u_0) + \varepsilon + \frac{1}{k}\{s_k(t)-s_0(t)\}. \tag{62}$$

Since $s_k(t)$ are positive and bounded uniformly in k, there is an integer K such that

$$\frac{1}{K}|s_K(T)-s_0(T)| < O(\alpha) \tag{63}$$

for all k such that $k \geqq K$. Then, from eq.(62) it follows that

$$\frac{1}{K}\sum_{i=0}^{K-1}\{2(1-\alpha)V_i(T)+U_i(\lambda;T)\} \leqq \alpha(1-\alpha)^2 Q(\lambda;\Delta u_0) + \varepsilon + O(\alpha). \tag{64}$$

Next we bear in mind that ε is sufficiently small and therefore the small forgetting factor α is larger than ε. We also note that $U_i(\lambda;t$ takes the guadratic form of r_i and $\dot{r}_i$ as defined by eq.(43) via eq.(59) and hence we see that

$$\begin{aligned} U_i(\lambda;t) &\geqq (1-\alpha)\int_0^t e^{-(\lambda+1)\tau}[r_i^T(\tau)Ar_i(\tau) \\ &\quad + \dot{r}_i^T(t)\{J+H(r_i(\tau)+q_d(\tau))\}\dot{r}_i(\tau)]d\tau \\ &= 2(1-\alpha)\int_0^t e^{-(\lambda+1)\tau}V_i(\tau)d\tau \end{aligned} \tag{65}$$

for $U(\lambda;r_k,\dot{r}_k)$ is nonnegative. Thus, we conclude from eq.(64) and

(65) that there is at least one k ($0 \leqq k \leqq K$) such that

$$2(1-\alpha)[V_k(T)+\int_0^T e^{-(\lambda+1)t} V_k(t)dt] \leqq U_k(\lambda;T)$$
$$\leqq \alpha(1-\alpha)^2 Q(\lambda;\Delta u_0) + \varepsilon + O(\alpha) \tag{66}$$

which results in

$$V_k(T) + \int_0^T e^{-(\lambda+1)t} V_k(t)d\tau \leqq \frac{1}{2}\alpha(1-\alpha)Q(\lambda;\Delta u_0)$$
$$+ \varepsilon/2(1-\alpha) + O(\alpha). \tag{67}$$

Inequality (67) implies that the exponentially weighted squared integration of r_k and $\dot{r}_k$ over $t \in [0,T]$ is of order α. More specifically, it holds

$$\int_0^T e^{-(\lambda+1)t} V_k(t)dt \leqq O(\alpha). \tag{68}$$

Then, it is possible to show the following:

<u>Lemma 2</u> Eq.(68) implies

$$\int_0^T e^{-(\lambda+1)t} \Delta u_k^T(t) \Phi^{-1} \Delta u_k^T(t)dt \leqq O(\alpha). \tag{69}$$

In this version of the paper, we omit the proof of the lemma which is lengthy and rather mathematically tactic, because presently we must show the generalized passivity of the dynamic equation concerning the difference $d_k = r_{k+1} - r_k$.

Once inequality (69) is ascertained, it is possible to refresh the content in the long-term memory by this input $u_k(t)$ and restart the training from i=K+1 (see Fig. 4). Then, the term $Q(\lambda;\Delta u_0)$ in eq.(60) becomes of order α and hence eq.(60) can be written in the form

$$s_{i+1}(t) \leqq (1-\alpha)s_i(t) + O(\alpha^2) + \varepsilon. \tag{70}$$

If $\varepsilon \leqq O(\alpha^2)$, then $s_i(t) \leqq O(\alpha)$ which eventually shows the existence of j in $K \leqq j \leqq 2K$ such that

$$\int_0^T e^{-(\lambda+1)t} V_j(t)dt \leqq O(\alpha^2). \tag{71}$$

Thus, it is possible to say that if such a specific learning scheme is adopted then

$$\lim_{i\to\infty} s_i(t) \leqq O(\varepsilon). \tag{72}$$

We call this learning scheme a strategic learning, which consits of the following steps:

1) Choose an initial command input $u_0(t)$ appropriately. For example, it is possible to set $u_0(t)\equiv 0$ (which means $\Delta u_0=-u_d$).

2) Repeat exercises until eq.(63) holds (say, i=K). At this step, it is important to prepare a stack memory in which the best input trajectory in the sense of the smallest value of the left hand side of eq.(66) is stored.

3) Read the input data $u_k(t)$ from the stack memory which must be the best among the trials from i=0 to i=1 and refresh the content of the long-term memory by this $u_k(t)$.

4) Restart exercises (Return to step 1)).

We now conclude that

Theorem 2 Under the same assumptions as those in Theorem 1, the strategic learning results in that motion trajectories approach an ε-neighborhood of the desired one and eventually remain in it.

IV. Conclusions

For a class of P-type learning control algorithms with a forgetting factor, robustness with respect to initialization errors, fluctuations of dynamics and measurement noises has been discussed. In particular, the uniform boundedness of motion trajectories has been proved by exploring the generalized possivity of the displacement dynamics. It has also been shown that output trajectories approach an ε-neighborhood of thedesired one and eventually remain in it provided a strategic learning introduced first in this paper is used.

Both Theorems 1 and 2 can be extended to the case of PI-type (Proportional and Integration) learning control.

References

[1] S. Arimoto, S. Kawamura, and F. Miyazaki: Bettering operation of robots by learning, Journal of Robotic Systems, Vol.1, pp.123-140, (1984).

[2] S. Kawamura, F. Miyazaki, and S. Arimoto: Iterative learning control for robotic systems, Proc. IECON '84, Tokyo, pp.393-398, (1984).

[3] S. Arimoto, S. Kawamura, and F. Miyazaki: Bettering operation of dynamic systems by learning; A new control theory for servomechanism and mechatronics systems, Proc. 23rd IEEE Conf. Decision and Control, Las Vegas, NV, pp.1064-1069, (1984).

[4] ibid: Can mechanical robots learn by themselves?, in "Robotic Research" The Second International Symposium, H. Hanafusa & H. Inoue, Eds., MIT Press, Cambridge, Massachusatts, pp.127-134, (1985).

[5] S. Arimoto: Mathematical theory of learning with applications to robot control, Proc. of 4th Yale Workshop on Applications of Adaptive Systems Theory, Yale University, New Haven, Connecticut, pp.379-388, (1985).

[6] S. Kawamura, F. Miyazaki, and S. Arimoto: Hybrid position/force control of manipulators based on learning method, Proc. '85 Inter Conf. on Advanced Robotics, Tokyo, Japan, pp.235-242, (1985).

[7] S. Arimoto S. Kawamura, F. Miyazaki, and S. Tamaki: Learning Control Theory for dynamical systems, Proc. 24th IEEE Conf. Decision and Control, Fort Lauderdale, Florida, pp.1375-1380,

(1985).

[8] S. Kawamura, F. Miyazaki, and S. Arimoto: Application of learning method for dynamic control robot manipulators, ibid, pp.1381-1386, (1985).

[9] ibid.: Realization of robot mation based on a learnig method, IEEE Trans. on Systems, Man, and Cybernetics, Vol.SMC-18, No.1, pp.126-134, (1988).

[10] S. Arimoto, S. Kawamura, and F. Miyazaki: Convergence, stability, and robustness of learning control schemes for robot manipulator, in M.J. Jamshidi, L.Y.S. Luh, and M. Shahinpoor (eds.), Recent Trends in Robotics: Modeling, Control, and Education, pp.307-316, Elsevier Sciencs Publishing Co., Inc., New York, (1986).

[11] G. Heinzinger, D. Fenwick, B. Paden, and F. Miyazaki: Robot learning control, Proc. 28th IEEE Conf. Decision and Control, Tampa, Florida, Dec. 13-15, (1989).

[12] S. Arimoto: Robustness of learning control for robot manipulators, Proc. of the 1990 IEEE International Conference on Robotics and Automation, pp. - Cincinnati, Ohio, May 13-18, (1990)

[13] C.G. Atkeson and J. McIntyre: Robot trajectory learning through practice, Proc. of 1986 IEEE International Conf. on Robotics and Automation, San Francisco, CA, (1986)

[14] S. Arimoto and F. Miyazaki: Stability and robustness of PID feedback control for robot manipulators of sensory capability, in "Robotic Research" The FirstInternational Symposium, by M. Brady & R.P. Paul, Eds., MIT Press, Cambridge, Massachusetts,

pp.783-799, (1984).

15] ibid.: Asymptotic stability of feedback control laws for robot manipulators, Proc. IFAC Symp. on Robot Control '85, Barcelona, Spain, pp.447-452, (1985).

16] D.E. Koditschek: Natural motion for robot arms, Proc. of 23rd IEEE Conf. Decision and Control, Las Vegas, NV, pp.733-755, (1987).

17] P. Bondi, G. Casalino, and L. Gambardella: On the iterative learning control theory for robotic manipulators, IEEE J. of Robotics and Automation, Vol.4, No.1, pp.14-22, (1988).

18] S. Arimoto: Learning control theory for robotic motion, to appear in International Journal of Adaptive Control and Signal Processing, (1990).

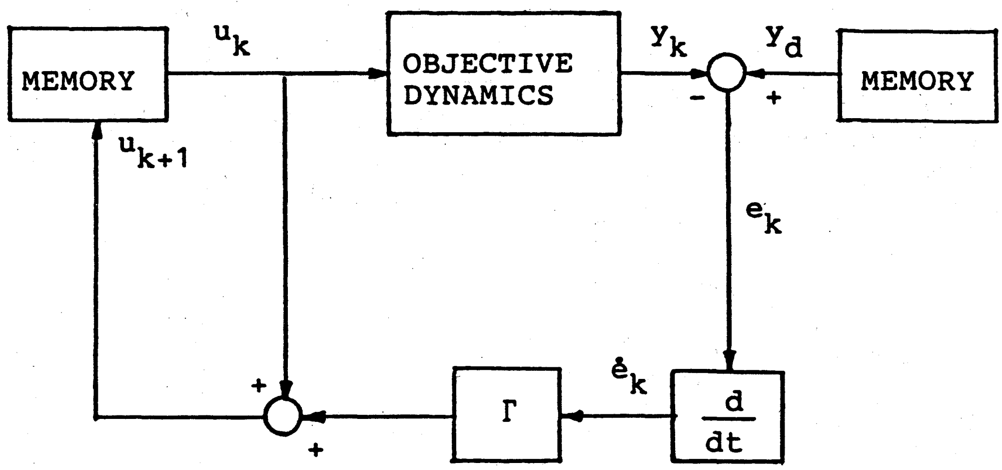

Fig.1 D-type learning control

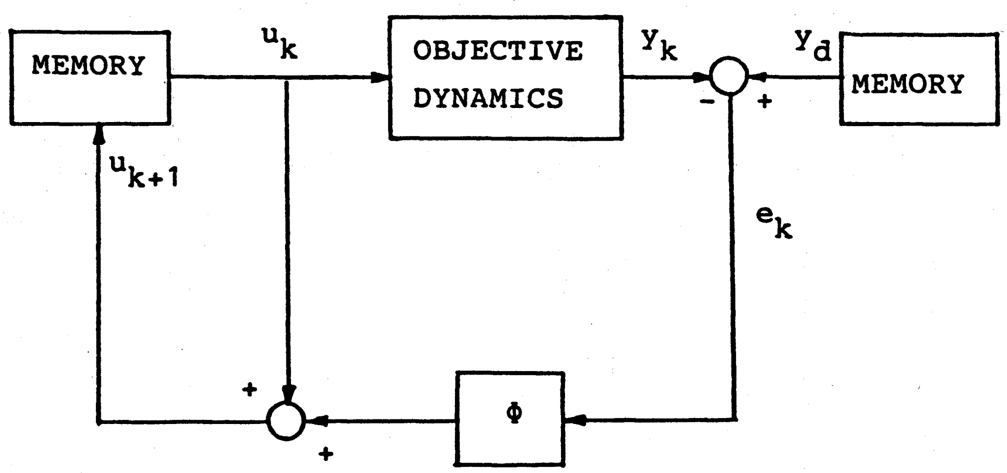

Fig.2 P-type learning control

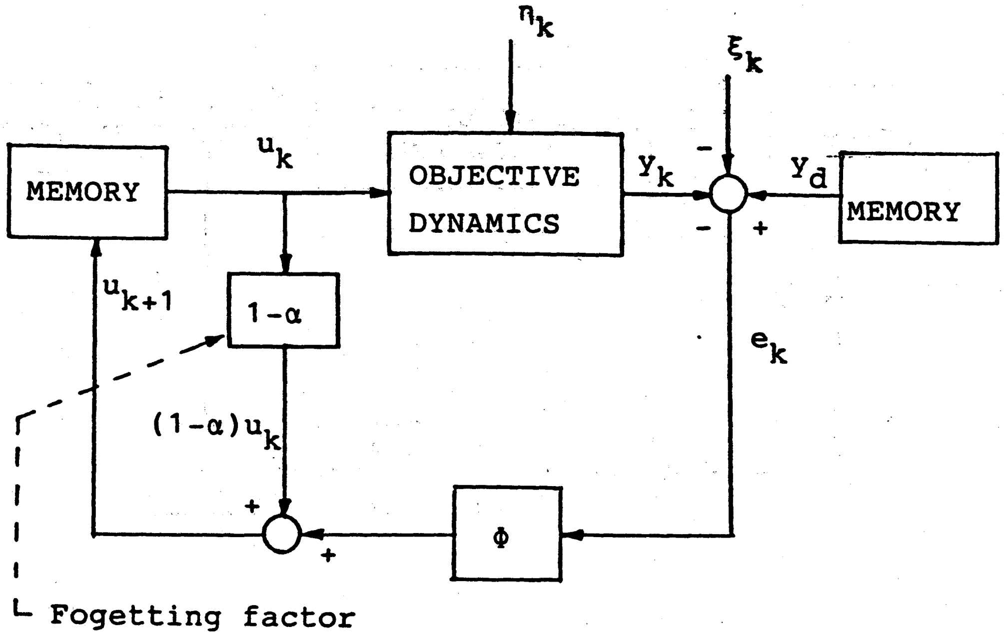

Fig.3 P-type learning control with a forgetting factor

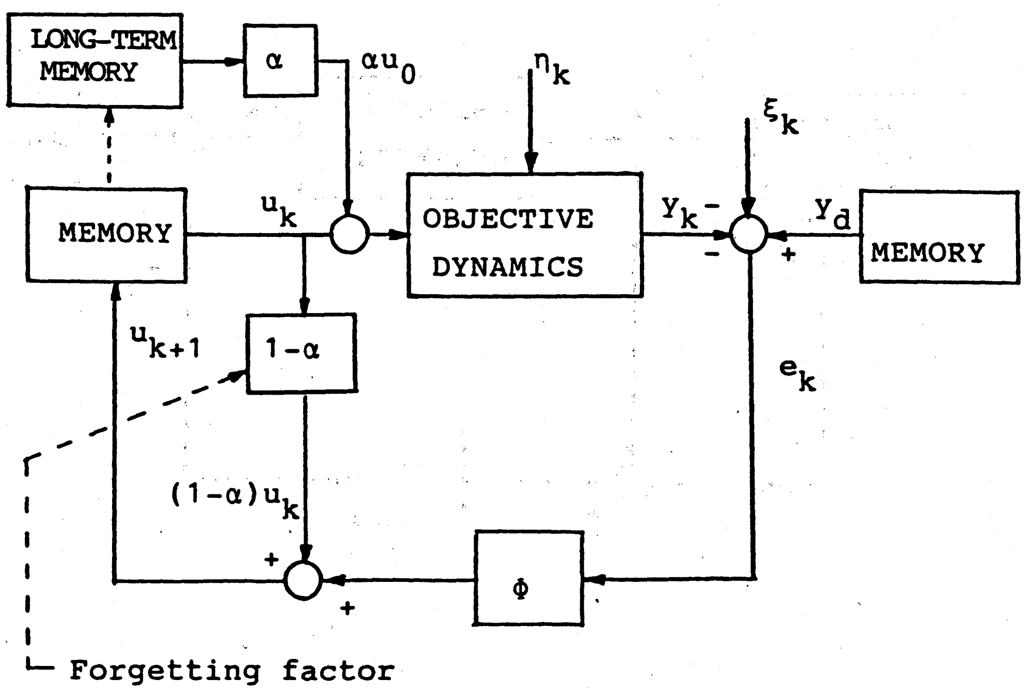

Fig.4 Refreshed P-type learning control with a forgetting factor